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Cooperation game: treasure digging.

There's a hidden treasure in a 2-dimensional square area that consists of grids. The length and width of the area can vary but is fixed during the game.
Before the game begins, the first player is told the x coordinate of the treasure and the second player the y coordinate.

The game progresses in steps of choosing *new* locations for digging. Both players announce in secret the location they're going to dig at, and the chosen locations are *revealed* and checked for the treasure existence after both players have made their decisions.

If the treasure is in the location a player has chosen, he wins. In case both find the treasure at the same time, it's a draw. The game continues until the treasure is found.

Question: what is the best strategy to choose locations?

Notes: If one player ignores what locations other one chooses, obviously the best strategy is to go through only those grid locations that his known partial coordinate matches with. E.g. If John knows the treasure is at $(x, 5)$, then he should go through $(1,5)$, $(2, 5)$, $(3, 5)$ and so on as long as the treasure in not yet found.
But if the other one is following John's decisions, he can conclude that John knows the treasure lies at a location where y-coordinate is 5. So with his own known x coordinate, the place is determined fully.

But on the other hand, both know this. So how much should one try to see what the other one does, and how much one should be trying to hide his own knowledge by selecting totally fake coordinates?

Variations:

How would this game change, if there were more players and the treasure world had as many dimensions as players?

How about if there was a limit for game steps, so that it is possible that neither one gets the treasure. Should they start cooperating if the players also allowed to negotiate during the game?

What if players couldn't dig from arbitrary place: they could move only to the neighbour grid in one game step and dig there. The start place could be chosen by the players (in secret). An additional rule: by not digging, you could move two grids instead of one.

Example game:

John knows x coordinate is 3.
Peter knows y coordinate is 2.

First round:
John chooses $(3, 1)$ (John uses his known partial coordinate)
Peter chooses $(1, 3)$ (Peter doesn't reveal his information)
Both misses.

Second round:
John chooses (3, 3) (Again John uses his knowledge, but tries to combine it with Peter's, which unfortunately for John was a fake.)
Peter chooses (3, 2) (Now, after faking his coordinate last turn, he guesses John used his own, and combines John's with his own to get the right solution.)
Peter wins the game.
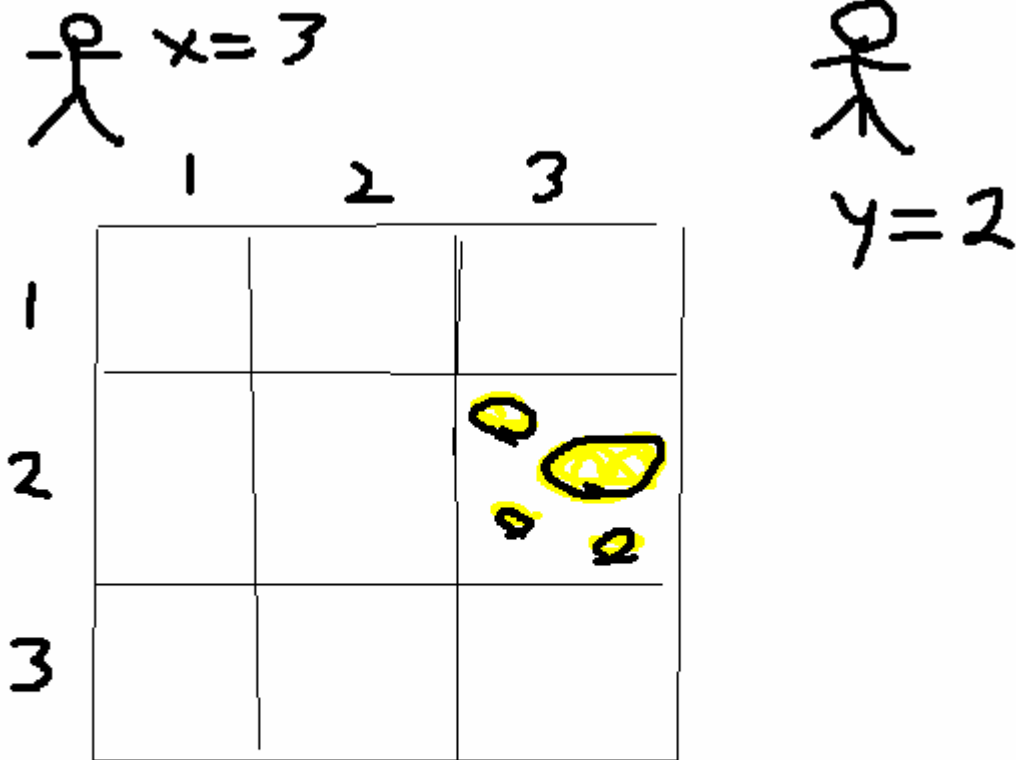
If both players choose wrong location in their first turn, should they use their opponent's last partial coordinate in their next guesses with his own known coordinate?